

# Safeguarding retirement in a bear market

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- What is the effect of ‘sequence-of-return’ risk—the risk of receiving a concentrated series of particularly poor returns—on retirees who depend on a financial portfolio to generate income? We provide a quantitative answer to this question by examining the cohorts that would have retired during or near six major US bear markets since 1926.
- Compared to otherwise similar investors retiring during the same periods, and assuming constant real-dollar withdrawals, the unlucky ones with a poor sequence of returns were 31% more likely to outlive their wealth, had 11% lower retirement income streams, and left 37% smaller bequests.
- These adverse effects can be mitigated with an adaptive withdrawal strategy. By countering a decline in portfolio value with an incremental decrease in planned withdrawal amounts, even those bearing the worst sequence-of-return risk could have eliminated the possibility of premature portfolio depletion and increased their bequests by 20%. These improvements would have required a manageable reduction in retirement income on the order of a 5% decrease in the first five years and effectively no change over the whole 35 years of retirement.

The growing reliance on defined contribution retirement accounts increasingly intertwines financial security in retirement with the investment risks of the public financial markets. Bear markets pose a unique challenge for new retirees, who must negotiate the interaction of portfolio withdrawals and poor returns. When markets are down in the first years of retirement, withdrawal strategies that would be prudent in most market environments can transform sequence-of-return risk into longevity risk—the risk of outliving a portfolio.

In this paper, we use historical return data on US equity and bond markets since 1926 to investigate the impact of sequence-of-return risk on investors who start their retirement during or near bear markets. We first seek to quantify the adverse effects when this risk is realised. Next, we attempt to understand the extent to which they can be mitigated.

Our analysis focuses on the interaction between retirement timing and withdrawal approach, and how sequence-of-return risk can lead to a wide range of outcomes. It assumes the investors retire with their entire wealth invested in a balanced portfolio evenly split between equities and bonds.

We further assume that the balanced portfolio is the sole source of income for the 35 years of retirement. Excluding other possible sources such as state benefits, defined benefit pension, employment income, and housing equity allows us to understand the impacts of sequence-of-return risk on liquid financial wealth—a critical and growing component of retirement portfolios. It also enables us to learn the extent to which adjustments in the withdrawal approach can lessen this risk after it is realised.

We begin by providing an empirical definition of sequence-of-return risk. We then introduce our strategy for quantifying the risk's impact on retirement outcomes and describe the metrics used. Finally, we quantify the impact when retirees follow either a constant real-dollar withdrawal approach or an adaptive one, illustrating how the latter can help ease the effects of a market decline.

### *Notes on risk*

*Investments are subject to market risk, including the possible loss of the money you invest. Past performance is no guarantee of future returns. Bond funds are subject to the risk that an issuer will fail to make payments on time and that bond prices will decline because of rising interest rates or negative perceptions of an issuer's ability to make payments.*

*Diversification does not ensure a profit or protect against a loss in a declining market. Performance data shown represent past performance, which is not a guarantee of future results. Note that hypothetical illustrations are not exact representations of any particular investment, as you cannot invest directly in an index or fund-group average.*

## Sequence-of-return risk when retiring in bear markets

Sequence-of-return risk is a concept first introduced by Bengen (1994), who examined historical financial market returns to identify sustainable portfolio withdrawal rates. The risk is most concerning when the start of retirement coincides with a poor return sequence, often during a bear market.

In these cases, every withdrawal turns a negative return, which is temporary in nature, into a permanent impairment of the balance. The amount withdrawn at a considerable loss reduces the opportunity to recover over the long term. Because the opportunity cost of withdrawing a dollar—measured in the expected return on the dollar until the end of retirement—is particularly high immediately after sequence-of-return risk strikes, the ‘magic’ of compounding return works against the retirees who must do so.

The varying degrees of sequence-of-return risk include, on one end of the spectrum, a long-term ‘generational luck’ aspect. This describes, for instance, the difference between investors who retire into either a decade-long bull market (as in the 1980s) or a decade of financial market turmoil (as in the 1970s) <sup>1</sup>.

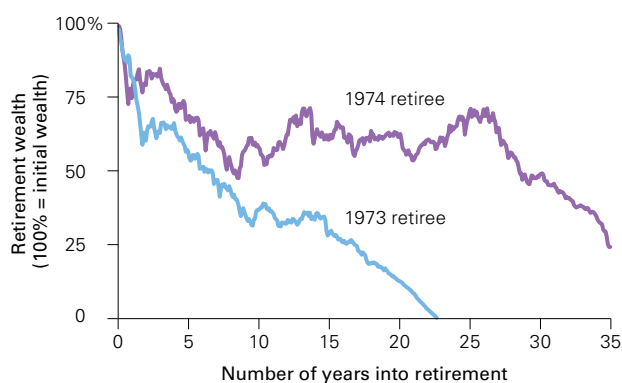
The other end of the spectrum contains a shorter-term ‘pure chance’ aspect. We refer here to a difference in retirement timing of one or two years—a difference that could have plausibly been altered for a host of reasons other than financial readiness.

Our study focuses on the latter type of sequence-of-return risk. As an example, consider two hypothetical investors, one retiring in 1973 and one in 1974, with a 35-year retirement horizon. Assume that both entered retirement with \$500,000 invested in a portfolio of 50% US equities and 50% US bonds, rebalanced monthly. Both planned to withdraw \$25,000 per year, adjusted for inflation.

Over the two 35-year periods, with 34 overlapping years, both investors could have earned broadly comparable long-term returns in the absence of withdrawals—5.23% real return per year for the 1973 investor, and 5.1% for the 1974 investor. However, with regular withdrawals of \$25,000 per year, adjusted for inflation, they would have experienced dramatically different outcomes (see Figure 1).

The 1973 retiree would have run out of money 23 years into retirement. The 1974 retiree’s portfolio, by contrast, would have maintained a balance of \$300,000 for most of the 35 years of retirement, finishing with a bequest equal to about a quarter of the pre-retirement amount. We attribute the different outcomes to sequence-of-return risk. The one-year difference in return (encompassing a severe bear market decline in 1973) considerably impaired the longevity of the 1973 retiree’s portfolio.

**Figure 1. Sequence-of-return risk: same generation, one-year return difference in retirement wealth over time**



**Note:** This figure assumes two hypothetical investors retiring in the beginning of the calendar year with \$500,000 portfolios invested 50/50 in equities and bonds and fixed withdrawal plans of \$25,000 per year (inflation-adjusted).

**Sources:** Vanguard calculations, based on data from Morningstar, Inc.; the Federal Reserve Bank of St. Louis; and the Kenneth R. French Data Library, available at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

<sup>1</sup> This is the type of sequence-of-return risk Bengen (1994) focused on in his study, which demonstrates that even when the financial markets produce similar returns over distinct long-term periods, the sequence of each period’s annual returns governs the sustainable withdrawal rate. Interest in this type of risk dates back to Samuelson (1969) in the academic literature. Viceira (2002) and Cocco et al. (2005) eventually incorporated realistic representations of retirement into the formulation of the inquiry.

## Outcomes for bear market and adjacent retirees

To quantify the impact of sequence-of-return risk systematically, we used monthly US equity market returns since 1926 and identified eight bear markets with at least 20% peak-to-trough declines that were accompanied or instigated by the prospect of a recession<sup>2</sup>. Using this definition, we identified 31 years in which our hypothetical investors would have retired during or near one of these markets.

We assumed that all commenced their retirement at the beginning of a calendar year (see **Figure 2**). We divided them into two groups—full bear market and partial bear market—based on the type of return sequence they would have faced upon retiring.

Full bear market retirees bear the brunt of a peak-to-trough decline. These include most who retire at a market's peak, as well as those who retire in the middle of a bear market.

Partial bear market retirees are those who retire at the tail end of a bear market—for example, in 1932 or 1938. They experience a generally rebounding market following a short decline. This group also includes those who retire a sufficient amount of time before the onset of a bear market—for example, in 1928 or 1961. They head into the remaining year or so of a bull market at the beginning of their retirement, which provides a cushion for the eventual decline.

**Figure 2. Full and partial bear market retirees**

Bear market	Full bear market	Partial bear market	Peak-to-trough decline (percentage)	Peak-to-trough duration (months)	Peak month	Trough month
1929	1929–1931	1928, 1932	83.7%	33	September 1929	June 1932
1937	1937	1936, 1938	49.3	13	February 1937	March 1938
1946	1946	1945, 1947	24.2	13	May 1946	June 1947
1962	1962	1961, 1963	23.0	6	December 1961	June 1962
1968	1968–1969	1967, 1970	33.6	19	November 1968	June 1970
1973	1973–1974	1972, 1975	46.4	21	January 1973	October 1974
2000	2000–2002	1999, 2003	45.1	31	March 2000	October 2002
2008	2007–2008	2006, 2009	50.4	17	October 2007	March 2009

Sources: Vanguard calculations, based on data from the Kenneth R. French Data Library.

<sup>2</sup> Major bear markets without a recession are rare; the Black Monday crash in 1987 is a notable exception because it was technical in nature, rather than driven by a prospective recession. All other major bear markets we have chosen took place in anticipation of or accompanying a recession.

By dividing the retirees into two groups—15 full and 16 partial—we establish a credible baseline for quantifying the effect of sequence-of-return risk on retirement outcomes. In each of the eight major bear markets identified in **Figure 2**, both types of retirees face the same economic events and market cycles and start their retirement during or near major bear markets, with the exception of one or two years at the start of retirement. This makes an ideal setting for us to determine the impact of sequence-of-return risk on long-term retirement outcomes for those retiring in recessionary/bear markets<sup>3</sup>.

We explore these retirees' outcomes along four dimensions:

**Risk of portfolio depletion.** The first and most important dimension is the likelihood of portfolio depletion over 35 years<sup>4,5</sup>.

**1. Size of portfolio withdrawals.** The second-most important dimension is the amount of income a cohort can generate in retirement.

**2. Magnitude and duration of portfolio value decline.** The third dimension includes a portfolio's maximum decline during retirement and the length of time it remains below its starting value (underwater duration). This dimension seeks to capture the experience of watching a portfolio's value diminish without knowing how long it must last.

**3. Size of bequest.** The fourth and final dimension is the value of the portfolio at the end of the 35 years; these assets can be left as a bequest or fund a longer retirement.

## Retiring into a bear market with a fixed spending rule

We first examine the outcomes for retirees using a 5% fixed spending strategy—a classic rule of thumb for determining the level of portfolio withdrawals. We assume that they retire with \$500,000 invested in a 50% US equity/50% US bond portfolio rebalanced monthly<sup>6,7</sup>. They plan to withdraw \$25,000 first year and adjust this amount each year for inflation.

Our analysis evaluates outcomes in real (inflation-adjusted) dollars to facilitate comparisons of periods with different inflation levels. We also apply a discount rate to future income of 1% per year to capture the preference for spending more today than tomorrow<sup>8</sup>. As a result, the present value of a 5% real withdrawal rate without portfolio depletion in our study is roughly \$737,000 instead of \$875,000.

Figures 3 through 5 show that the outcomes for those who retire into or near bear markets with this spending rule are not encouraging<sup>9</sup>. Both groups face a risk of exhausting their portfolios, and on average, their retirement income falls short of the spending rule's implied level.

<sup>3</sup> Generally, retiring investors would not know which of the two groups they belong to. We compare them to shed light on the potential maximum downside impact of sequence-of-return risk for full bear market retirees using a fixed spending strategy and on how much of that impact might be mitigated by an adaptive withdrawal strategy.

<sup>4</sup> We assume that retirees will live an additional 35 years, so that a person retiring at the age of 66 would live to 101. Of course, not all will survive this long. The ways in which evolving health and financial conditions affect mortality are outside the scope of this paper.

<sup>5</sup> Because the requirement of 35 years of return data limits which of the eight bear markets and their affiliated cohorts we can study, we leave out those who retired during or shortly after the 2000 and 2008 bear markets.

<sup>6</sup> Returns on US equities are from the Kenneth R. French Data Library; intermediate-term US government bond returns are from Morningstar's *Yearbook on Stocks, Bonds, Bills, and Inflation*; and US inflation data is from the *CPI Index for All Urban Consumers*. All data are monthly and for the period July 1926 to December 2019.

<sup>7</sup> We also assume no fees or taxes, which could potentially change various effects shown in this paper.

<sup>8</sup> This captures a time preference—the preference to consume now rather than tomorrow, all else being equal—and is distinct from an adjustment related to mortality risk.

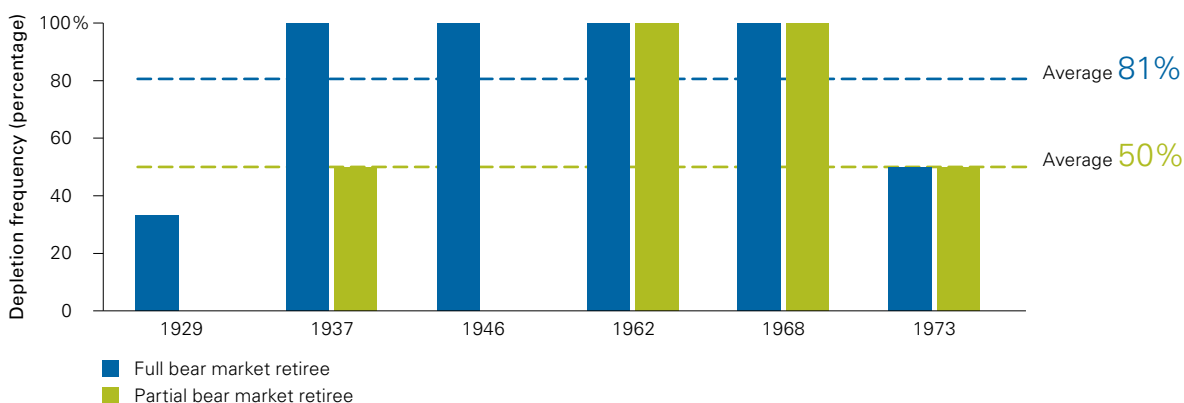
<sup>9</sup> Since the seminal observation by Bengen (1994) that a 4% fixed spending rule would have been a safe choice for the historical period ended in 1992, the literature has evolved to acknowledge the merits of a wide variety of alternative withdrawal strategies with an adaptive component. Our choice to first consider the impact of sequence-of-return risk under the 5% fixed spending rule is motivated by our interest in obtaining the worst-case outcomes.

More striking than the generally disappointing outcomes, however, is the difference between the two groups. In a clear illustration of sequence-of-return risk, 81% of the full bear market retirees depleted their wealth (see Figure 3a). Their retirement income, on average,

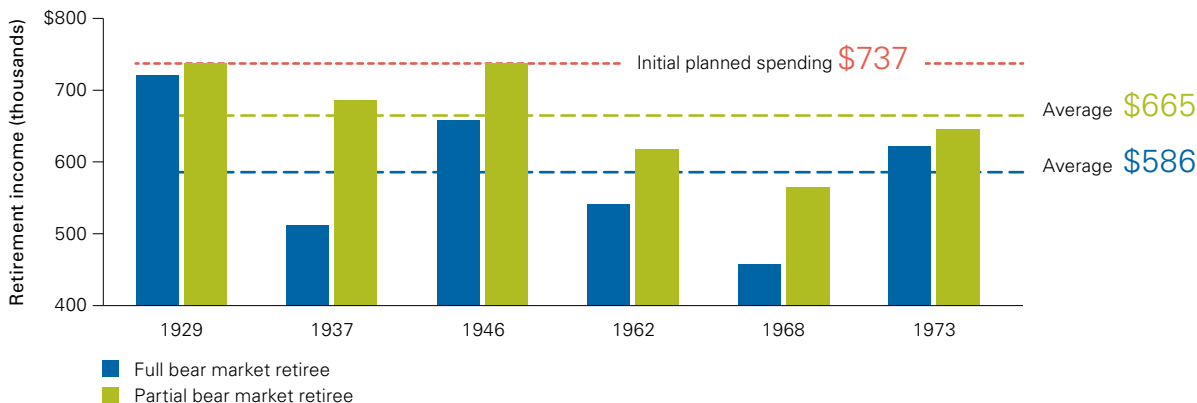
fell short of the originally planned level by 21% (Figure 3b). Their partial bear market retiree counterparts, by contrast, had a portfolio depletion rate of 50% and a retirement income shortfall of 10%<sup>10</sup>.

Figure 3. Depletion and retirement income under fixed spending

a. Frequency of depletion in 35 years



b. Retirement income over 35 years



**Notes:** These graphs show depletion and retirement income for hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds and a fixed withdrawal plan of \$25,000 per year (inflation-adjusted) for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

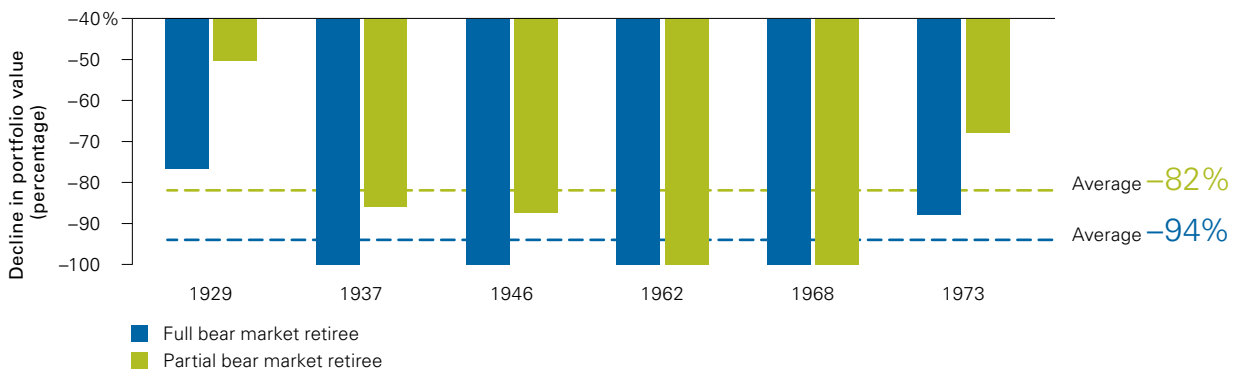
<sup>10</sup> The early depletion of both types shows that the averages of any retirement are influenced by the years when depletion took place. This observation provides a valuable perspective as we examine the differences in outcomes under other metrics such as maximum decline and bequests.

The other two dimensions also highlight the costly impact of sequence-of-return risk. On average, the partial bear market retiree's portfolio value experienced a maximum decline of 82%, compared with 94% for the full bear market retiree (see Figure 4a). The latter's

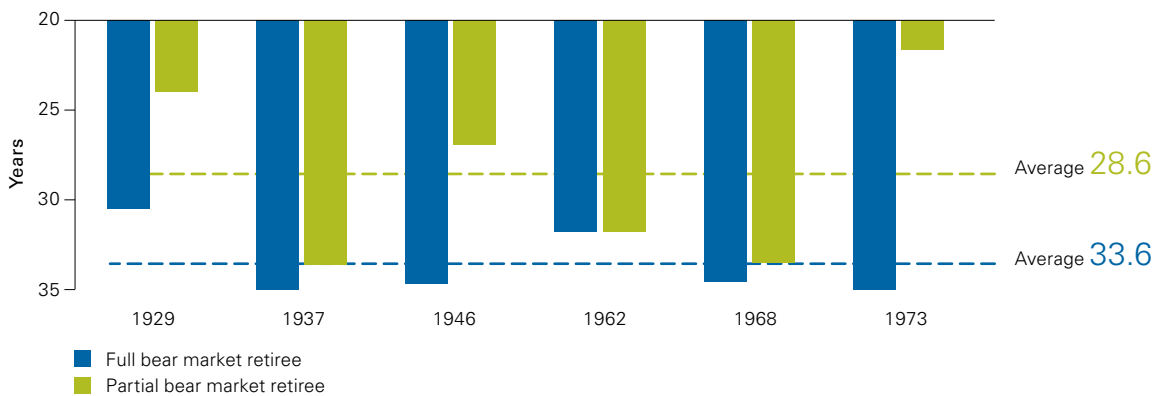
portfolio almost never returned to its initial value—it was underwater for 34 years (Figure 4b). By contrast, the partial bear market retiree's portfolio remained underwater, on average, for 29 years.

**Figure 4. Maximum portfolio decline and underwater duration under fixed spending**

a. Maximum decline in portfolio value



b. Underwater duration



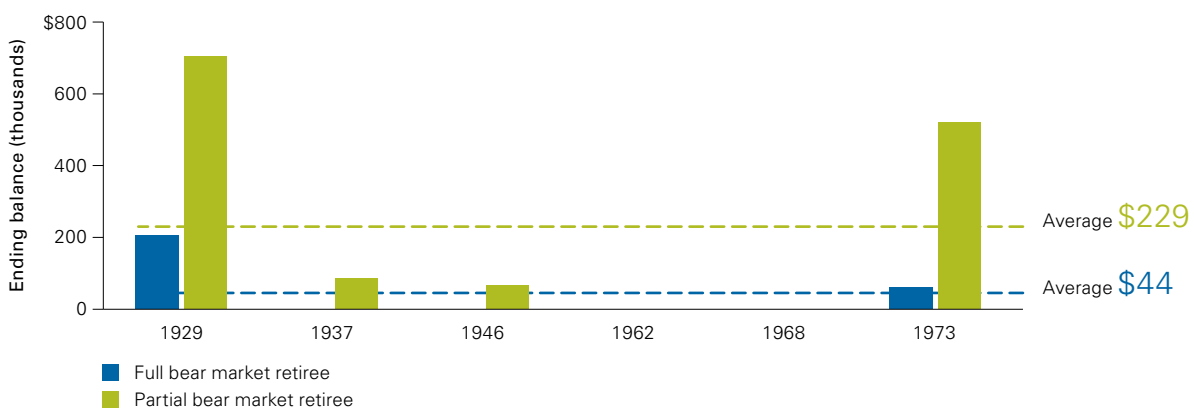
**Notes:** These graphs show the maximum decline in portfolio value and underwater duration for hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in stocks and bonds and a fixed withdrawal plan of \$25,000 per year (inflation-adjusted) for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

The full bear market retirees finished the 35-year period with an average portfolio value of \$44,000 available for a bequest, and the partial bear market retirees with \$229,000. While both averages were pulled down by the 1962 and 1968 bear markets, when retirement portfolios were prematurely depleted for both types of bear market cohorts, all other bear markets show a stark difference in ending balance between the two groups (see Figure 5).

Because our analysis uses a relatively high fixed spending rule applied to some of the worst economic and market periods in the past century, it is not surprising that the outcomes for all of the bear market retirees are disappointing. Even so, the marginal differences between the average full bear market retiree and the average partial bear market retiree, driven by a one-year difference in retirement date, underscore the impact of a poor sequence of returns on new retirees.

Figure 5. Ending portfolio value under fixed spending



**Notes:** This figure shows the ending portfolio value for hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds and a fixed withdrawal plan of \$25,000 per year (inflation-adjusted) for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.



### Dynamic spending: a response to sequence-of-return risk

Next, we explore a strategy to manage sequence-of-return risk. We stress-test a newer withdrawal strategy called 'dynamic spending'<sup>11</sup> to examine the extent to which a sensible adaptive approach would have helped a portfolio weather the worst recessionary bear markets of the past and changed the relative differences in outcomes for the two types of bear market retirees. We evaluate these outcomes along the same four dimensions used in the previous section.

The key to managing poor returns early in retirement using a dynamic spending approach is to reduce portfolio withdrawals. When markets recover, withdrawals can be ratcheted higher. As in the fixed spending rule case, we set an initial spending target of \$25,000, or 5% of the portfolio's initial value. The next year, we multiply the portfolio's value by 5% and compare that figure with the previous year's spending. If the new one is higher, we increase it up to a predetermined ceiling, such as a 5% increase from the previous year.

If the new figure is lower, we reduce the spending, but not below a predetermined floor, such as a 2% reduction from the previous year's level. There is

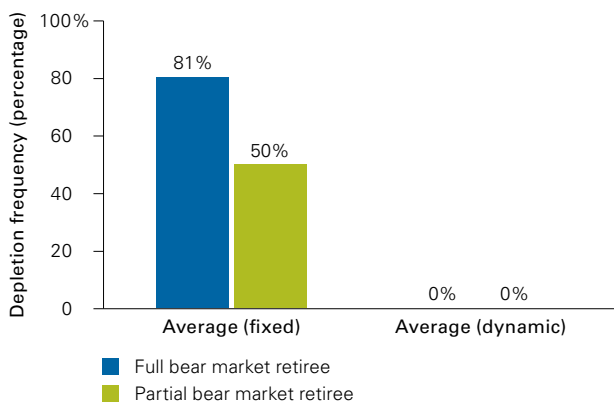
room for customisation in the ceiling and floor. Jaconetti et al (2020) found that the lower the floor, the greater the likelihood of not depleting retirement wealth. In the analysis that follows, we set the ceiling and floor at a 5% increase and a 2% decrease in real terms from the prior year's level<sup>12</sup>. We repeat this process every year, reducing or increasing withdrawals based on changes in portfolio value.

This strategy eliminates the risk of portfolio depletion for all bear market retirees, even those who retired into the Great Depression and the tumultuous late 1960s and 1970s (see **Figure 6a**). These periods presented the greatest risks in our almost-century's-worth of data. For both groups, overall retirement income remained largely at the same level.

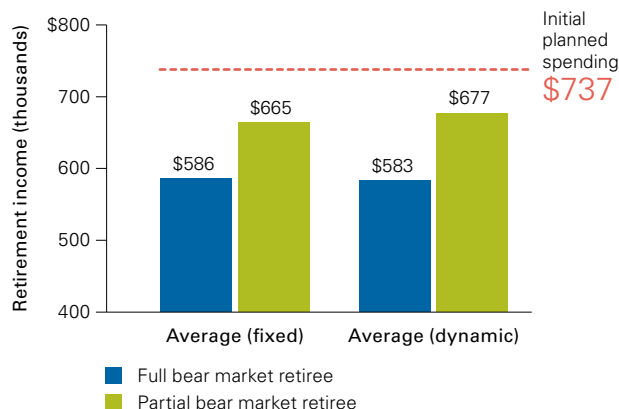
Most retirees we studied had challenging generational luck, reducing their lifetime income. For full bear market retirees, the income available under dynamic spending is 21% below the initially planned amount of \$737,000. For partial bear market retirees, lifetime income was 8% below this amount. However, combined with no depletion, these outcomes represent a marked improvement over the ones shown in Figure 4 under a fixed spending approach.

**Figure 6. Depletion and retirement income under dynamic spending**

a. Frequency of depletion in 35 years



b. Retirement income over 35 years



**Notes:** These graphs show depletion and retirement income for hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds. Our study compared fixed spending and dynamic spending withdrawal approaches over the 35-year retirement horizon.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

<sup>11</sup> For a nonexhaustive list, we refer the reader to the works of Guyton (2004), Milevsky and Robinson (2005), Stout and Mitchell (2006), Blanchett and Frank (2009), and Jaconetti et al. (2020). While the dynamic spending strategies proposed in this literature are diverse in both concept and implementation, they are in agreement that the rigidity of a fixed spending approach can be unhelpful to some retirees. In this paper, we follow the general strategy proposed by Jaconetti et al. (2020).

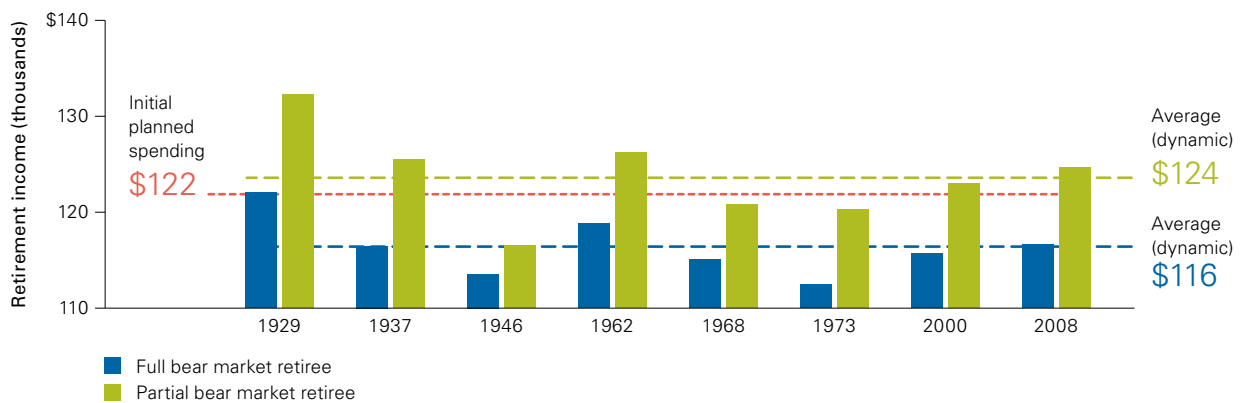
<sup>12</sup> We considered other configurations of ceilings and floors, including a symmetric setup of 3% for both. The results in this paper are qualitatively robust in comparison.

We now turn our attention to the austerity required by a dynamic spending strategy. In the first five years of retirement (see **Figure 7a**), the average full bear market retiree experiences a modest income shortfall of 4.5%, while the average partial bear market retiree has an income surplus of 2%.

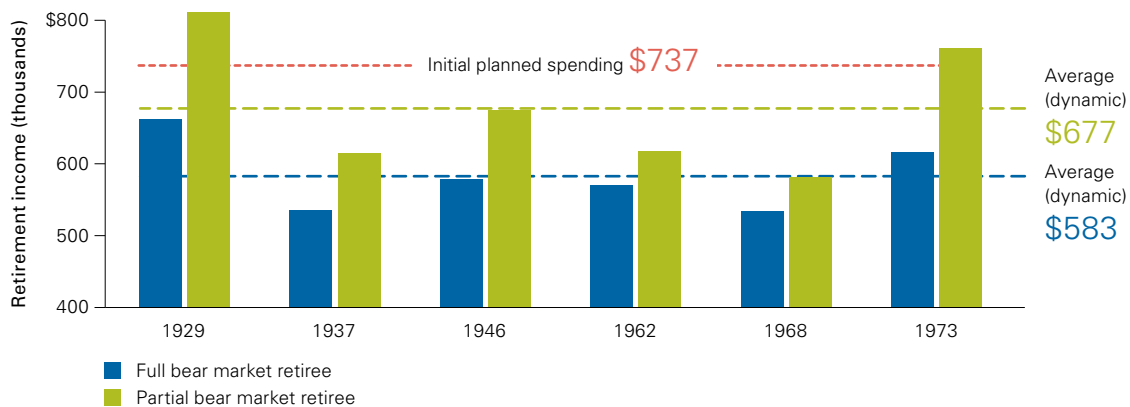
The majority of partial bear market retirees are able to spend more than initially planned in the first five years because dynamic spending revises spending up in tandem with the rebounding market returns.

**Figure 7. Retirement income under dynamic spending: short term and long**

a. Short term (five years)



b. Long term (35 years)



**Notes:** These graphs show the ending portfolio value of hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds and a dynamic spending plan for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

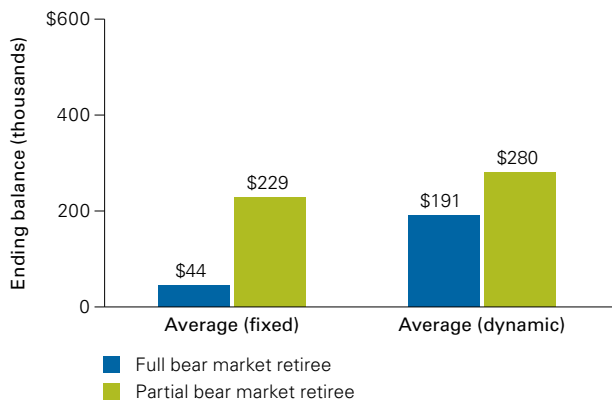
The approach of introducing a gradual reduction in income over the longer term leaves more of the portfolio invested, so that it benefits from an eventual market rebound. Particularly for full bear market retirees, this translates into more wealth left at the end of the retirement.

The average full bear market retiree's ending portfolio balance increased from a mere 9% of its initial value under fixed spending to 38% under the dynamic

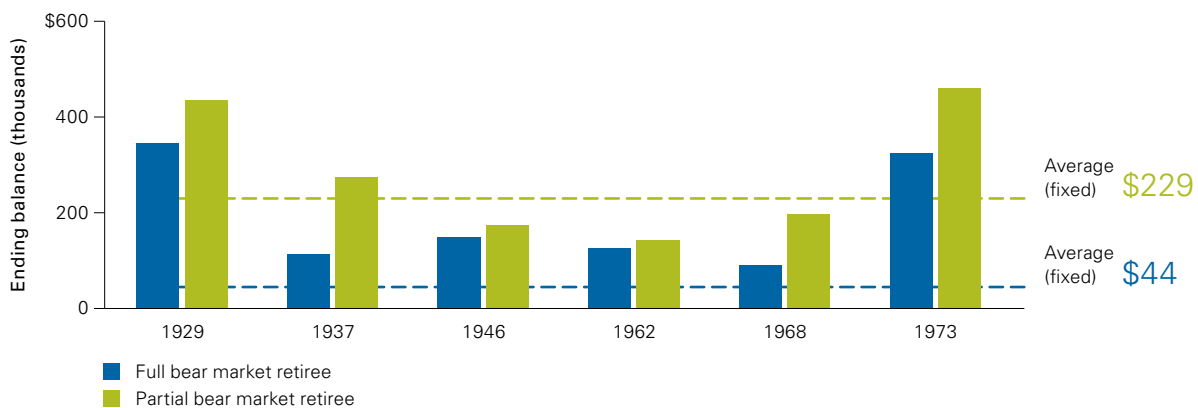
approach—a 29-percentage-point improvement (see **Figure 8a**). The average partial bear market retiree's ending balance increased a more moderate 12 percentage points. This markedly reduced gap between the two groups represents a significant buffer against the prospect of depletion, offering the affected full bear market retirees greater peace of mind.

**Figure 8. Ending portfolio value under dynamic spending**

a. Comparison with fixed spending



b. Dynamic spending during historical bear markets



**Notes:** These graphs show the ending portfolio value of hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds and a dynamic spending plan for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

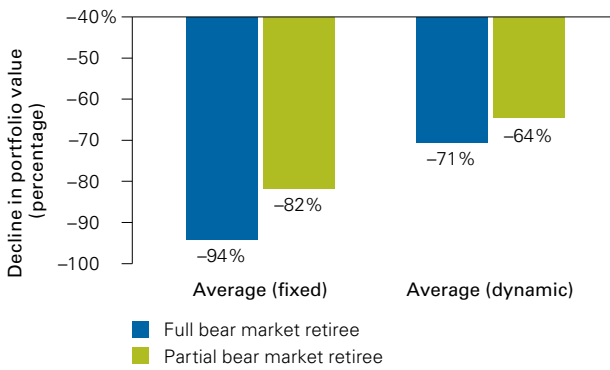
The overall wealth of both types of retirees also declined less in the worst market environment. Whereas the wealth of the average full bear market retiree dropped 94% under the fixed spending rule, it fell only 71% under dynamic spending (see **Figure 9a**). Even partial bear market retirees experienced a 22% improvement, indicating again that dynamic spending helps bridge the large gap in ending wealth.

Much of the difference in outcomes between the two groups under fixed spending can be mitigated by dynamic spending (as shown in Figures 6 through 9),

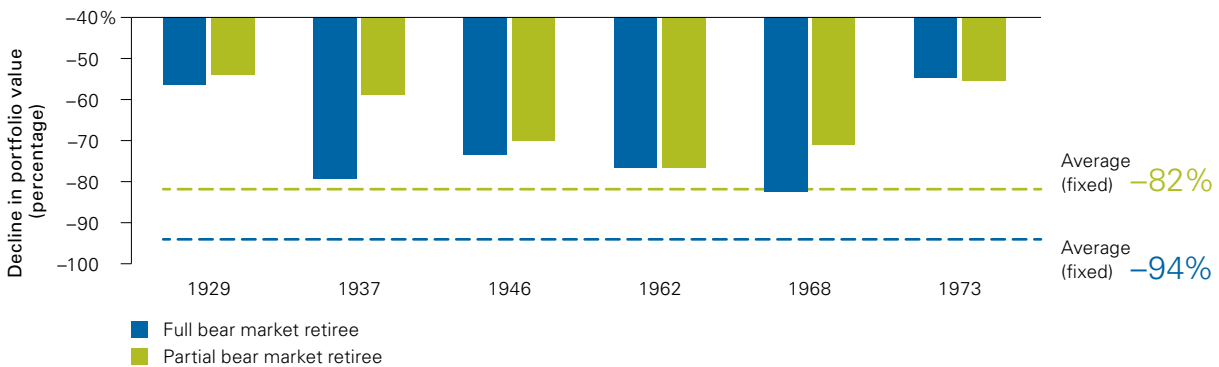
which helps preserve a greater portion of wealth after a poor sequence of returns. Both groups experience less depletion risk, a reduction in maximum portfolio value decline, and an increase in ending wealth. However, because an adverse sequence of returns has a more severe effect on full bear market retirees under a fixed spending rule, dynamic spending improves their outcomes disproportionately. Thus, they bridge much of the gap in outcome between themselves and partial bear market retirees.

**Figure 9. Maximum portfolio decline under dynamic spending**

a. Comparison with fixed spending



b. Dynamic spending during historical bear markets



**Notes:** These graphs show the maximum decline in portfolio value of hypothetical investors retiring during or near six historical recessionary bear markets. Each investor retired at the beginning of the calendar year with a portfolio of \$500,000 invested 50/50 in equities and bonds and a dynamic spending plan for a 35-year retirement.

**Sources:** Vanguard calculations, based on data from Morningstar, Inc., the Federal Reserve Bank of St. Louis, and the Kenneth R. French Data Library.

## Conclusion

We used the worst recessionary bear markets of the past century to explore the impact of sequence-of-return risk on the withdrawal stream that can be generated from a portfolio of financial assets in retirement. We compared the outcomes of those who retired into the worst periods with those whose retirement date differed by a year or two. We determined that, for those relying on a fixed real-dollar withdrawal strategy, this small difference at the beginning of retirement can lead to significant differences in long-term outcomes. These include a greater risk of portfolio depletion, a lower level of lifetime retirement income, and a smaller amount for bequests.

We also demonstrated how an adaptive withdrawal strategy—dynamic spending—can help limit the downside risk of portfolio depletion. It can reduce much of

the difference in bequests between the unlucky cohort that retires in the worst years of a bear market and its generational peers. These improvements require a modest reduction in income stream in the short run and effectively no change over a 35-year retirement in comparison to using the fixed real-dollar withdrawal approach.

Our focus on the investment portfolio excluded other sources of retirement income such as state benefits and defined benefit pensions. The integration of a full range of income sources is beyond the scope of this paper. For instance, retirees with annuitised sources of income will likely need less withdrawal adaptiveness than we examined in this paper. Our study nevertheless provides useful insights on the risks of retiring into a bear market and strategies to manage these risks.

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